

CONTENTS

Abstract	iv
Problem Status	iv
Authorization	iv
INTRODUCTION	1
CONCERNING THE "SIMPLE" SERIES MAGNETIC AMPLIFIER	2
A NEW SERIES AMPLIFIER	14
A NEW PARALLEL-CONNECTED MAGNETIC AMPLIFIER	21

ABSTRACT

A new theory of magnetic amplifier operation, developed at this Laboratory, is presented. An elementary quasi-mathematical approach is used to demonstrate the application of this theory to known circuits. The method of analysis is also used to demonstrate a few of the new circuits which have been predicted by utilization of this new theory. These circuits have characteristics which are, in many ways, far superior to those of the magnetic amplifier circuits in use today.

PROBLEM STATUS

This is an interim report; work on this problem is continuing.

AUTHORIZATION

NRL Problems E03-22R, E03-26D, and E04-29R
NS 678-054, NE 130-706, NR 424-290

ON THE MECHANICS OF MAGNETIC AMPLIFIER OPERATION

INTRODUCTION

There have been many analyses of magnetic amplifiers presented in the literature, many showing remarkably close check with experimental results. But there still seems to be room for a discussion of the mechanics of amplifier operation as nearly devoid of mathematical elegance as possible. This paper is essentially an effort to "talk through" an analysis of magnetic amplifier operation. More exacting mathematical analyses based upon the concepts presented herein are being prepared by this Laboratory and will be published as they are concluded.

In order to provide understanding rather than a method of general mathematical solution, a semidescriptive or quasi-mathematical technique is utilized. Since this problem is quite complex, "exact" mathematical prediction seldom leads to a clear picture. This report is designed to give such a picture of the physics of magnetic amplifier operation and to give examples of the usefulness of the fundamental concepts. It will seem light reading to many of our mathematically minded scientists — that is the author's aim. It is hoped that this presentation will answer many of the questions raised by the engineers in this field who seldom have the time available to analyze the general solutions published in the literature. A further object of this report is the introduction of examples of certain new magnetic amplifier circuits developed as a direct result of this study.

In this report certain magnetic amplifier circuits will be illustrated and their circuit equations presented subject to certain assumptions. The equations will be interpreted in as clear a manner as the author thinks possible and some conclusions will be drawn. Since the mathematics of analysis of magnetic amplifiers is quite complex as viewed from continuous theory, a clearer period-by-period interpretation will be pursued during which periods the equations may be considered as a continuous, elementary, and reasonably accurate representation of the phenomena. The two periods which will be used are those (a) during which the amplifier output current is large (conducting period) and (b) during which the amplifier output current is of the order of the magnetizing current (nonconducting period). Equivalent circuits for each period will be set up. The final conditions of one period will represent the initial conditions of the following period.

The two assumptions utilized in this analysis are (a) ideal magnetization curves for the core materials (Figure 1), and (b) ideal rectifiers. The best core materials and rectifiers manufactured today rather closely approximate these assumed conditions. A resistive load impedance will be used for simplification of the mathematics.

The conclusions drawn from these elementary considerations will form a basis for predicting the performance of examples of certain novel circuits which will have properties of an unusual nature. Practical discussion of these circuits is beyond the scope of this report and will be presented elsewhere.

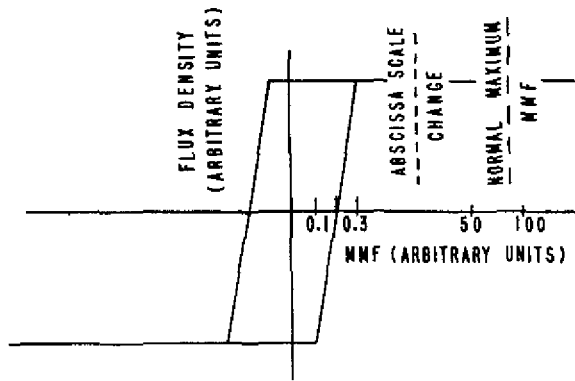


Figure 1 - Ideal magnetization curve

A relation will be derived (in an elementary manner) between the response time of magnetic amplifiers and the factors which influence this phenomenon. A rectified sinusoidal signal will be used to avoid the consideration of certain transcendental equations which would arise in the case of a dc signal. The resulting expression will be a sequence whose terms will approach a final value in the n th term. The number n will directly measure the response time, and the final value in the sequence will be the steady-state firing angle for the output current.

There will be presented a discussion of the function of the control voltage in magnetic amplifier operation. It will be shown that the control source need never supply the power for the amplifier control. The energy storage in the amplifier cores in steady-state operation can be supplied entirely from the ac power source. The result is a very efficient amplifier with response time of approximately one-half cycle. Remarks regarding power gain will be difficult to interpret since the control source will never be required to supply any power to the amplifier circuit.

This approach to the magnetic amplifier problem has resulted from a recognition of the fact that the magnetic amplifier is a voltage-sensitive device and not, as generally believed, a current-sensitive device. The only truly independent variable is the control voltage. Of course control means which are "current sources" provide exceptions to these remarks.

CONCERNING THE "SIMPLE" SERIES MAGNETIC AMPLIFIER

Two Transformers and One Transformer

The simple series amplifier shown schematically in Figure 2 consists of a pair of single-phase transformers I and II connected in series in the primary or output circuit and energized from an ac source E_{ac} in series with a load circuit impedance R_L . The secondary or control circuit has the two transformer secondaries connected in series opposition (for ac voltages of fundamental frequency) in series with a control circuit resistance R_c and a control voltage source E_c .

The equations for this circuit configuration are:

$$E_{ac} = E_I + E_{II} + I_L R_L \quad (\text{Primary Circuit}) \quad (1)$$

$$NE_I = NE_{II} + E_c - I_c R_c \quad (\text{Secondary Circuit}) \quad (2)$$

where E_I and E_{II} are the primary voltages on transformers I and II respectively, I_L and I_c are the primary and secondary currents respectively, and N is the ratio of secondary turns to primary turns.

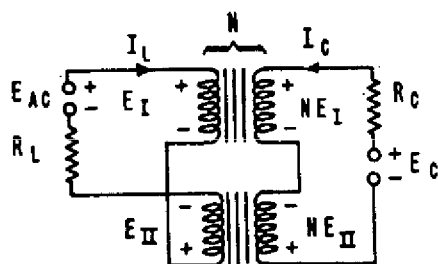


Figure 2 - Series magnetic amplifier

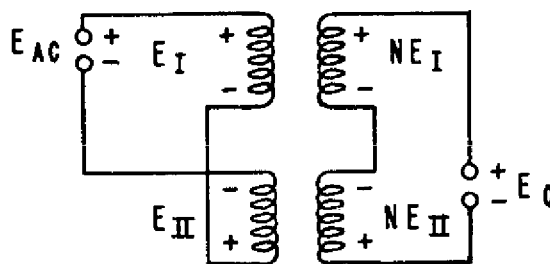


Figure 3 - Equivalent circuit for nonconducting period (core I proceeding to saturation)

However, during the periods of time during which transformer cores I and II are unsaturated simultaneously, currents I_L and I_C are assumed quite negligibly small (Figure 1). During these periods the circuit equations become from (1) and (2):

$$E_{ac} = E_I + E_{II} \quad (\text{Primary Circuit}) \quad (3)$$

$$NE_I = NE_{II} + E_C \quad (\text{Secondary Circuit}) \quad (4)$$

This will be called the nonconduction period. The equivalent circuit for this period is shown in Figure 3.

Whenever either transformer core is saturated, the primary and secondary voltages of that transformer ideally cease to exist and currents I_L and I_C can no longer be neglected. If core I is saturated, the circuit equations become from (1) and (2):

$$E_{ac} = E_{II} + I_L R_L \quad (\text{Primary Circuit}) \quad (5)$$

$$0 = NE_{II} + E_C - I_C R_C \quad (\text{Secondary Circuit}) \quad (6)$$

The output current I_L must equal the reflected control current NI_C , for core II remains unsaturated and can have no net ampere turns beyond that needed for magnetization. This period of time will be called the conducting period. The equivalent circuit for this period is shown in Figure 4. If core II is saturated (negative half-cycle of ac voltage) the circuit equations become from (1) and (2):

$$E_{ac} = E_I + I_L R_L \quad (\text{Primary Circuit}) \quad (7)$$

$$0 = -NE_I + E_C - I_C R_C \quad (\text{Secondary Circuit}) \quad (8)$$

where I_L equals $(-)NI_C$. Another equivalent circuit like that of Figure 4 would redundantly represent this.

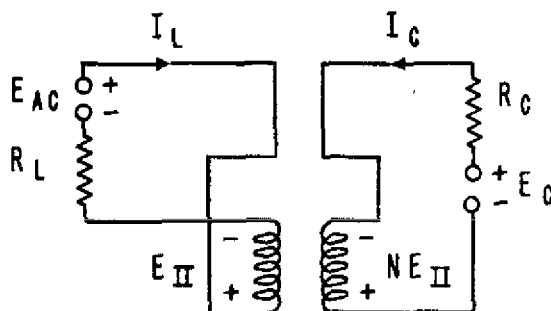


Figure 4 - Equivalent circuit for conducting period (core I saturated)

A Matter of Timing

To say that the equations listed above are the only ones to be observed would hardly tell the whole story. One must determine when each is to be used. In setting up the equations it was stated that one set of equations was for the periods during which both cores were unsaturated simultaneously (nonconducting period) and the other set was for the periods during which either one of the cores was saturated (conducting period). It is clear from the above considerations that the flux level in both cores must be known at all times if the equations are to be useful.

There exists only one fundamental relation between magnetic flux and the electrical quantities one can conveniently measure. This relation between rate of change of flux and applied voltage was first published by Faraday and has been subsequently called Faraday's Law of Electromagnetic Induction. It is this relation which will be used here.

Since the equations being used in this report concern only voltages and currents it is deemed advisable for clarity to keep this magnetic flux measurement in the same units. This will avoid the confusion associated with the introduction of a magnetic quantity. The integral of the voltages E_I and E_{II} will measure flux changes in their respective cores. The transformer core will be saturated when the integral of its terminal voltage attains a value in volt-seconds equivalent to that which may be stored by the transformer. A convenient measure of this value is obtained from the ac source voltage E_{ac} which time integral over $\frac{1}{2}$ a half-cycle is equivalent to the volt-second storage of both cores. That is to say $\frac{1}{2} \int_0^{\frac{1}{2f}} E_{ac} dt$ will be the volt-second capacity of one transformer core on a traverse of flux from its negative maximum value to its positive maximum value. The term $\frac{1}{2f}$ is the time of one half-cycle of ac supply voltage where f is the line frequency in cycles per second. These amplifiers are regarded as being supplied from an ac source which voltage is the maximum that can be applied and still keep the cores from saturating at any time due to ac alone.

Since $I_L = NI_C$ elimination of E_{II} from (5) and (6) gives

$$\left(E_{ac} + \frac{E_C}{N}\right) = \left(\frac{R_C}{N^2} + R_L\right) I_L \quad (9)$$

From this relation it is apparent that the output current will lag the ac voltage by an angle dependent upon the voltages E_{ac} and E_C when E_C is a constant dc voltage. This lag can be eliminated by utilization of a rectified ac voltage for E_C whose frequency and phase is the same as that of the ac source. This type of control voltage is in very wide use and is almost indistinguishable from dc control with the same average voltage. This step results in quite a simplification when one recognizes that the initial instant of each period of operation is now clearly defined. The conducting period begins when one of the cores saturates; the nonconducting period begins when the ac voltage reaches zero.

In all problems involving definite integrals some set of initial conditions must be specified — a starting point. The author has chosen the no-load condition ($E_C = 0$) sufficiently long after application of the ac voltage E_{ac} so that all transients due to the application of

ac have ceased to exist. At this time the amplifier is operating according to (3) and (4) with $E_I = E_{II}$, $E_c = 0$ and both E_I and E_{II} simultaneously equal to $1/2 E_{ac}$. Since the cores are identical and the two transformers are in series, the magnetizing currents are identical and flow in the primary circuit.

The flux in the cores is swinging from knee to knee on the magnetization curves 90° behind the ac voltage wave (i.e., maximum flux deviation at zero applied voltage) and the magnetization current is exactly that indicated by the magnetization loops. These relations are shown in Figures 5 and 6. When E_{ac} is zero and going positive the line current is zero and the fluxes in the cores are at points A. Subsequently, the ac voltage becomes positive, the current I_L assumes the values at B and continues to increase according to the magnetization loop (proportional to Φ) until E_{ac} approaches zero at which time I_L attains the value at C and becomes zero at D. During the succeeding negative half-cycle the line current assumes the values at E, F, and when E_{ac} becomes zero, A. Since the magnetization loops are identical, $I_L + NI_c$ and $I_L - NI_c$ are equal and $I_c = 0$ with $E_c = 0$ no matter how "fat" the magnetization loop. A current I_c can only flow when there is a difference in magnetization level between the two cores and therefore the magnetization currents are carried by the primary circuit as the quiescent current. This is a reasonably accurate picture of the relations existing with $E_c = 0$.

The exact instant of application of E_c will be designated as point A, when E_{ac} is zero and E_{ac} is positive — the beginning of a nonconducting period. Time will be called zero at this instant and will attain a value $1/2f$ second at the end of the half-cycle; at the beginning of each half-cycle time measurement will be zero.

The Magnetizing Voltages

Equations (3) through (8) show the voltage and current relations existing in the simple series magnetic amplifier in nonconducting and conducting states. From these equations expressions for the magnetizing voltages E_I and E_{II} may be derived.

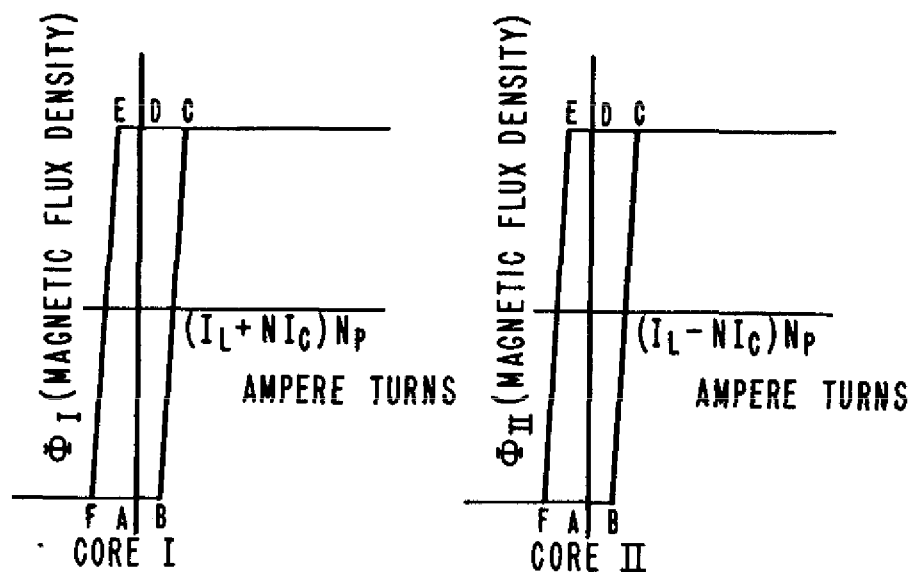


Figure 5 - Magnetization loops showing instantaneous MMF for $E_c = 0$

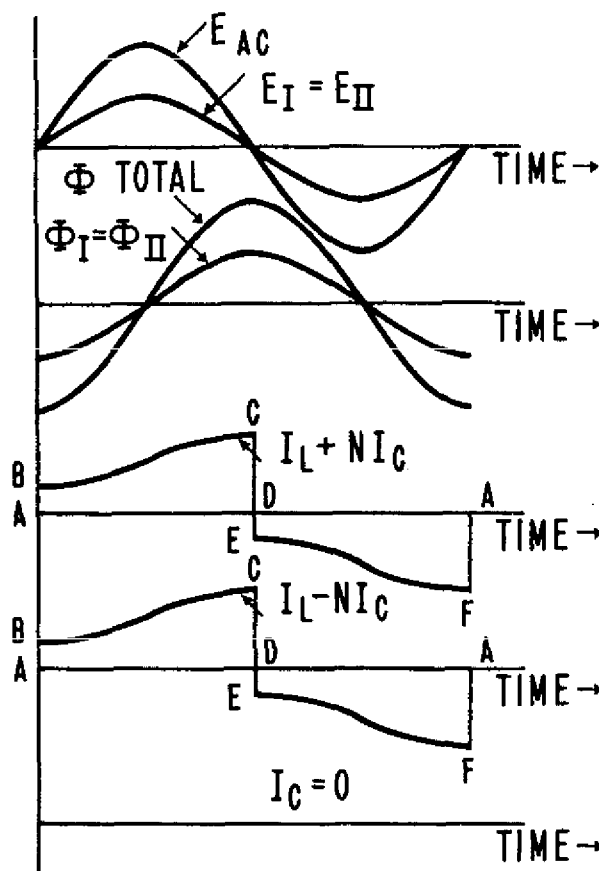


Figure 6 - Wave shapes of voltage, magnetic flux, and current with $E_c = 0$

During periods of nonconduction (3) and (4) are applicable. Solution of these equations gives:

$$E_I = \frac{1}{2} \left(E_{ac} + \frac{E_c}{N} \right)$$

$$E_{II} = \frac{1}{2} \left(E_{ac} - \frac{E_c}{N} \right)$$

Since all subsequent calculations will involve only a half-cycle at a time, it will be convenient to work with magnitudes without regard to direction. The equations for the nonconducting period become:

$$|E_I| = \frac{1}{2} \left[|E_{ac}| + \frac{|E_c|}{N} \right] \text{ (Core I proceeding to saturation),}$$

$$|E_{II}| = \frac{1}{2} \left[|E_{ac}| - \frac{|E_c|}{N} \right] \text{ (Core II deviating from saturation),}$$

during the positive half-cycles. During the negative half-cycles:

$$|E_I| = \frac{1}{2} \left[|E_{ac}| - \frac{|E_c|}{N} \right] \text{ (Core I deviating from saturation),}$$

$$|E_{II}| = \frac{1}{2} \left[|E_{ac}| + \frac{|E_c|}{N} \right] \text{ (Core II proceeding to saturation).}$$

During periods of conduction with core I saturated (positive half-cycle of ac voltage) Equations (5) and (6) yield:

$$|E_{II}| = \frac{|E_{ac}| - \frac{NR_L}{R_c} |E_c|}{1 + \frac{N^2 R_L}{R_c}}.$$

In summary:

- (1) Either core proceeding to saturation will have :

$$|E_I| = |E_{II}| = \frac{1}{2} \left[|E_{ac}| + \frac{|E_c|}{N} \right] \text{ volts} \quad (10)$$

across its primary terminals in such a direction as to produce the desired saturation.

- (2) Either core deviating from saturation during a nonconducting period will have:

$$|E_I| - |E_{II}| = \frac{1}{2} \left[|E_{ac}| + \frac{|E_c|}{N} \right] \quad \text{volts.} \quad (11)$$

- (3) Either core deviating from saturation during a conducting period will have:

$$|E_I| - |E_{II}| = \frac{|E_{ac}| + \frac{NR_L |E_c|}{R_c}}{1 + \frac{N^2 R_L}{R_c}} \quad \text{volts} \quad (12)$$

across its primary terminals in the appropriate direction.

Since the subsequent calculations will begin and end within a half-cycle the use of magnitudes will not prove cumbersome. The symmetry of the expressions for E_I and E_{II} during conducting and nonconducting periods will, however, prove extremely convenient.

Concerning Transients

In the preceding discussion a review of the circuit equations of the series magnetic amplifier indicated two modes of operation could be expected, and for each of these modes a simple equivalent circuit could be used. It has further been determined that the times of validity of these equivalent circuits could be definitely and conveniently established. Convenient initial conditions have been designated and described.

The problem has been set up in this manner to illustrate the fundamental physics involved. Since the magnetic amplifier is transient by nature any discussion disregarding this fact would fail to present a clear picture of its operation. To understand what goes on in "steady-state" operation one must also clearly understand the transient operation.

The following discussion will picture period by period the operation of the magnetic amplifier in transient until the general term is recognized.

During the first half-cycle of ac voltage immediately following the application of E_c (3) and (4) are applicable and will remain effective until core I reaches saturation. This will occur at a time t_1 defined by the relation:

$$\int_0^{t_1} E_I dt = \frac{1}{2} \int_0^{\frac{1}{2T}} E_{ac} dt = \Delta_0 \quad \text{volt-seconds.}$$

The value E_I is determined from (10). There follows in the interval (t_1 to $1/2t$) a period of conduction during which the voltage across transformer I (E_I) is zero and the currents I_L and I_c are determined by (5) and (6). Core II in the first half-cycle has E_{II} across its terminals causing it to deviate from its saturated value by an amount:

$$\Delta_1 = \int_0^{t_1} E_{II} dt + \int_{t_1}^{\frac{1}{2f}} E_{II} dt \quad \text{volt-seconds.}$$

The value of E_{II} for the nonconducting period (0 to t_1) is defined by (11) and for the conducting period (t_1 to $1/2f$) by (12). The resultant changes in the core fluxes are shown in Figure 7. This pictures the mechanism of the first (positive) half-cycle of ac supply voltage.

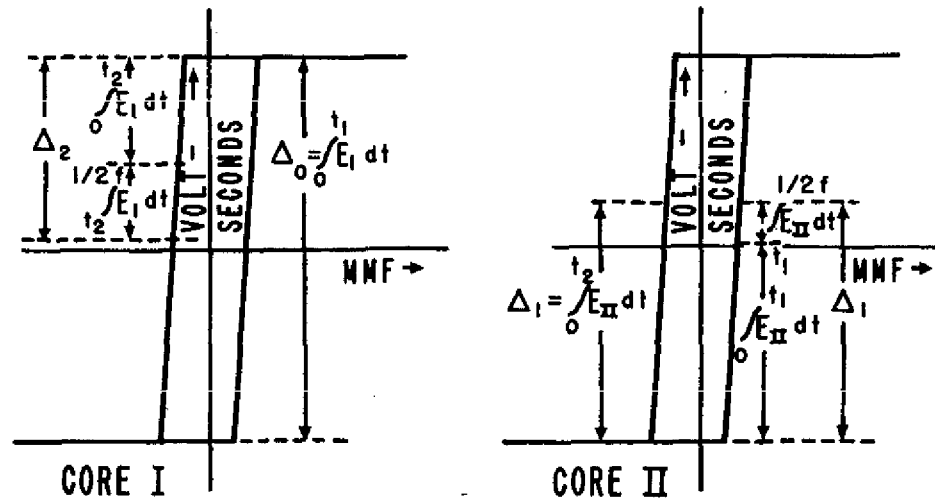


Figure 7 - Magnetization changes during first cycle after initiation of transient

During the second (negative) half-cycle of ac supply voltage core II will saturate at a time t_2 defined by:

$$\int_0^{t_2} E_{II} dt = \Delta_1 \quad \text{volt-seconds.}$$

That is, in order to saturate, core II must return to its saturated value and the only mechanism available is by releasing the volt-seconds absorbed during the previous half-cycle. After saturation E_{II} becomes zero and a conducting period ensues until the ac voltage reaches zero.

Core I meanwhile has E_I across its terminals causing it to deviate from its saturated value by an amount:

$$\int_0^{t_2} E_I dt + \int_{t_2}^{\frac{1}{2f}} E_I dt = \Delta_2 \quad \text{volt-seconds,}$$

where E_I is defined according to (11) in the nonconducting interval (0 to t_2) and by (12) in the conducting interval (t_2 to $1/2f$). The resultant changes in the core fluxes are shown in

Figure 7. This pictures the mechanics of the second (negative) half-cycle of ac supply voltage.

During the third (positive) half-cycle of ac voltage the steps of the first half-cycle are repeated with the same equations effective. The only change is in the initial conditions, i.e., instead of being $\frac{1}{2} \int_0^{\frac{1}{2T}} E_{ac} dt$ volt-seconds from saturation, core I is Δ_2 volt-seconds from saturation. The saturation will occur at a time t_2 defined by:

$$\int_0^{t_2} E_I dt = \Delta_2 \quad \text{volt-seconds.}$$

Or, more generally, saturation in any odd $(n-1)$ st half-cycle will occur at a time t_{n-1} defined by:

$$\int_0^{t_{n-1}} E_I dt = \Delta_{n-2} \quad \text{volt-seconds.}$$

Core II meanwhile has E_{II} across its terminals and will deviate from its saturated value in any odd $(n-1)$ st half-cycle by an amount:

$$\int_0^{t_{n-1}} E_{II} dt + \int_{t_{n-1}}^{\frac{1}{2T}} E_{II} dt = \Delta_{n-1} \quad \text{volt-seconds.}$$

During the succeeding even (n) th half-cycle, saturation will occur at time t_n defined by:

$$\int_0^{t_n} E_{II} dt = \Delta_{n-1} \quad \text{volt-seconds.}$$

Or:

$$\int_0^{t_n} E_{II} dt = \int_0^{t_{n-1}} E_{II} dt + \int_{t_{n-1}}^{\frac{1}{2T}} E_{II} dt.$$

However E_I and E_{II} have the same magnitudes during each respective part of a half-cycle [(10), (11), and (12)]. A general expression defining t_n for any n th half-cycle can be found by substituting those values for E_I and E_{II} in the equation above. This fundamental relation for transient operation is:

$$\frac{1}{2} \left[\left| \hat{E}_{ac} \right| + \frac{\hat{E}_c}{N} \right] \int_0^{t_n} \sin \omega t dt = \frac{1}{2} \left[\left| \hat{E}_{ac} \right| - \frac{\left| \hat{E}_c \right|}{N} \right] \int_0^{t_{n-1}} \sin \omega t dt + \frac{\left| \hat{E}_{ac} \right| - \frac{NR_L}{R_c} \left| \hat{E}_c \right|}{1 + \frac{N^2 R_L}{R_c}} \int_{t_{n-1}}^{\frac{1}{2T}} \sin \omega t dt.$$

Since \hat{E}_{ac} is a known constant the bracketed coefficients may be replaced by the following functions of \hat{E}_c :

$$f_1(\hat{E}_c) = \frac{1}{2} \left[\left| \hat{E}_{ac} \right| + \frac{\left| \hat{E}_c \right|}{N} \right],$$

$$f_2(\hat{E}_c) = \frac{1}{2} \left[\left| \hat{E}_{ac} \right| - \frac{\left| \hat{E}_c \right|}{N} \right],$$

$$f_3(\hat{E}_c) = \frac{\left| \hat{E}_{ac} \right| \frac{NR_L}{R_c} \left| \hat{E}_c \right|}{1 + \frac{N^2 R_L}{R_c}}$$

This gives:

$$f_1(\hat{E}_c) \int_0^{t_n} \sin \omega t \, dt = f_2(\hat{E}_c) \int_0^{t_{n-1}} \sin \omega t \, dt + f_3(\hat{E}_c) \int_{t_{n-1}}^{\frac{1}{2T}} \sin \omega t \, dt. \quad (13a)$$

When the definite integrals are evaluated one finds:

$$\cos \omega t_n = \frac{-f_2(\hat{E}_c) - f_3(\hat{E}_c) + f_1(\hat{E}_c)}{f_1(\hat{E}_c)} + \frac{f_2(\hat{E}_c) - f_3(\hat{E}_c)}{f_1(\hat{E}_c)} \cos \omega t_{n-1}$$

or in terms of the saturation angle θ_n

$$\cos \theta_n = 1 - f_3(\hat{E}_c) + f_4(\hat{E}_c) \cos \theta_{n-1} \quad (n \geq 1) \quad (13)$$

where $f_5(\hat{E}_c) = \frac{f_2(\hat{E}_c) + f_3(\hat{E}_c)}{f_1(\hat{E}_c)}$, $f_4(\hat{E}_c) = \frac{f_2(\hat{E}_c) - f_3(\hat{E}_c)}{f_1(\hat{E}_c)}$. The saturation angle

for the first half-cycle may be calculated from (13a): $\cos \theta_1 = -f_2(\hat{E}_c)/f_1(\hat{E}_c)$.

These preceding equations show that the saturation time or "firing angle" is directly determined by the previous saturation time and the magnitude of the signal voltage. From the general expressions above one can construct the transient output current as a function of time after application of a signal voltage. Current "firing" will occur in any n th half-cycle at the time t_n and the current will have instantaneous values determined by (9) for the remaining part of the half-cycle. The angle at which saturation occurs will change according to (13) until such time as θ_n equals θ_{n-1} ; this will be the steady-state condition. The "response time" could be defined as that number of cycles needed for the firing angle to reach some predetermined part of its final value.

Of Currents and Time

Since firing occurs at angle θ determined by (13) and the magnitude of current is determined by (9), calculation of current for transient and steady-state conditions is a relatively easy problem. The average current for any n th half-cycle is

$$\begin{aligned}
 I_L(\text{av}) &= \frac{1}{\pi} \int_{\theta_n}^{\pi} I_L d\theta = \frac{1}{\pi} \int_{\theta_n}^{\pi} \frac{\hat{E}_{ac} + \frac{E_c}{N}}{\frac{R_c}{N^2} + R_L} d\theta = \frac{1}{\pi} \frac{\hat{E}_{ac} + \frac{E_c}{N}}{\frac{R_c}{N^2} + R_L} \left[1 + \cos \theta_n \right] \\
 &= f_5(\hat{E}_c) \left[1 + \cos \theta_n \right] \quad (\text{transient and steady-state average current}). \quad (14)
 \end{aligned}$$

That is, for any given amplifier, the output current is directly proportional to $(1 + \cos \theta_n)$ and to a function of \hat{E}_c . For any given set of initial conditions $f_5(\hat{E}_c)$ would become a constant and $(1 + \cos \theta_n)$ would be directly proportional to the averaged output current. The value $\cos \theta_n$ can be obtained from (13) for transient conditions so it is possible to plot the average output current as a function of time from known initial conditions. Steady-state firing angle θ_{ss} can be determined from (13) by setting $\theta_n = \theta_{n-1} = \theta_{ss}$. This will give:

$$\cos \theta_{ss} = \frac{1 - f_5(\hat{E}_c)}{1 - f_4(\hat{E}_c)} \quad (15)$$

relating the steady-state firing angle θ_{ss} and the circuit parameters. By substitution of this expression in place of θ_n in (14) one gets

$$I_L(\text{av}) = f_5(\hat{E}_c) \left[1 + \frac{1 - f_5(\hat{E}_c)}{1 - f_4(\hat{E}_c)} \right] \quad (\text{steady-state average current}).$$

Substitution of the appropriate expressions for the functions of \hat{E}_c gives

$$I_L(\text{av}) = \left(\frac{2}{\pi} \right) N \frac{\hat{E}_c}{R_L} = N \frac{E_c(\text{av})}{R_c}, \quad (16)$$

i.e., the average load ampere turns equal the average control ampere turns in steady-state operation.

As an example, a magnetic amplifier is analyzed in transient, using (14). The values: $\hat{E}_{ac} = 100$ volts, $N = 1/2$ and $R_L = 50$ ohms are chosen as circuit parameters. For purposes of comparison several values of R_c were chosen and the control voltage E_c was adjusted to give a constant final value of 90° for θ_n ; $(1 + \cos \theta_n)$ equal to unity. Steady-state current firing at 90° is accomplished by setting $\cos \theta_n$ equal to $\cos \theta_{n-1}$, and both equal to zero in (13). This gives: $1 - f_5(\hat{E}_c) = 0$, the relation between E_c and R_c for 90° firing. Initial conditions are made the same as described in subsection B. The plot of these transients of $(1 + \cos \theta_n)$ is shown in Figure 8. The time of response is measured by the time required for $(1 + \cos \theta_n)$ to reach 63% of its final value (shown as the dashed line in Figure 8).

Direct calculation of response time of the series magnetic amplifier appears to be a step-by-step process when the general solutions are involved. However, considerable simplification of the mathematics may be attained by specifying convenient test conditions. A steady-state firing angle of 90° will prove convenient since the expression $(1 - f_5(\hat{E}_c))$

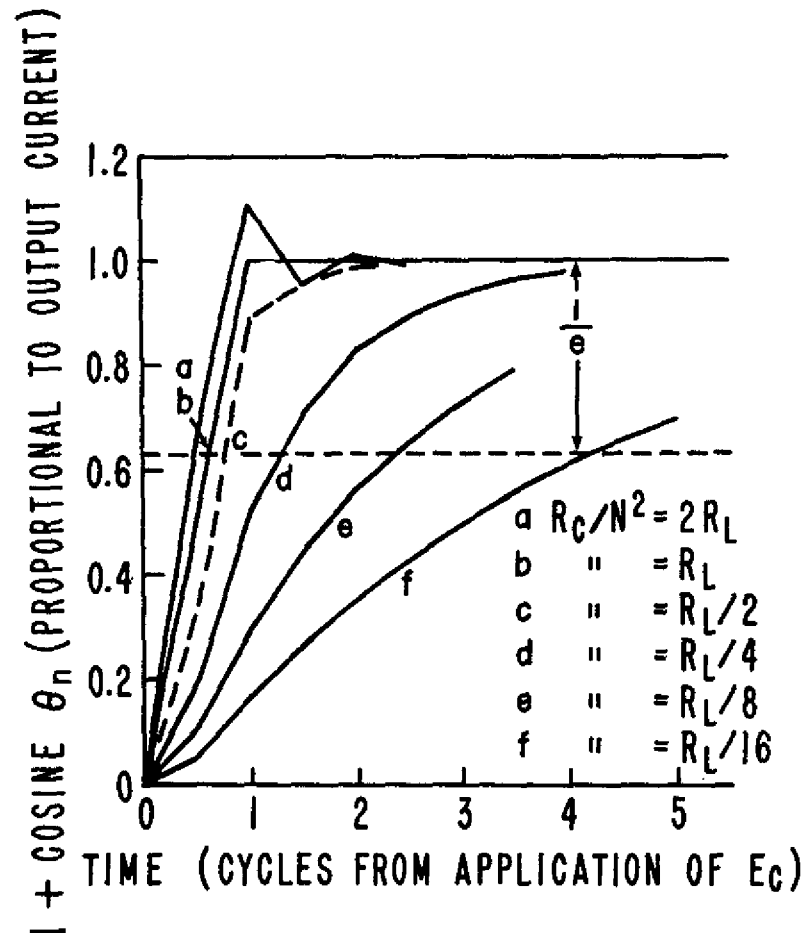


Figure 8 - Response characteristics, series magnetic amplifier

of (13) becomes zero. Since the time response of the simple series magnetic amplifier is considered to be independent of the applied signal voltage this simplification seems to be completely justified. So (13) becomes:

$$\cos \theta_n = f_s(\hat{E}_c) \cos \theta_{n-1} \quad (n > 1) \quad (17)$$

with the condition: $1 - f_s(\hat{E}_c) = 0$, wherein E_c and R_c are always adjusted so that the steady-state firing angle is 90° .

Substitutions of the firing angle θ_n for each succeeding n th half-cycle in (14) gives:

$$I_{L1(av)} = f_s(\hat{E}_c) \left[1 - \left(\frac{f_2(\hat{E}_c)}{f_1(\hat{E}_c)} \right) \right] \quad (\text{first half-cycle}),$$

$$I_{L2(av)} = f_s(\hat{E}_c) \left[1 - f_s(\hat{E}_c) \left(\frac{f_2(\hat{E}_c)}{f_1(\hat{E}_c)} \right) \right] \quad (\text{second half-cycle}),$$

$$I_{L_3}(\text{av}) = I_0(\hat{E}_c) \left[1 - I_4^2(\hat{E}_c) \left(\frac{I_2(\hat{E}_c)}{I_1(\hat{E}_c)} \right) \right] \text{ (third half-cycle),}$$

or generally:

$$I_{L_n}(\text{av}) = I_0(\hat{E}_c) \left[1 - I_4^{n-1}(\hat{E}_c) \left(\frac{I_2(\hat{E}_c)}{I_1(\hat{E}_c)} \right) \right] \text{ (nth half-cycle).}$$

The final steady-state current will be from Equation (17):

$$I_{L_{ss}}(\text{av}) = I_0(\hat{E}_c) \left[1 + I_4(\hat{E}_c) \cos 90^\circ \right] = I_0(\hat{E}_c).$$

The time of response is that time at which the output current reaches $(1 - 1/e)$ of its final value so the expression determining the n th half-cycle at which the output current reaches this quantity is

$$\frac{I_{L_n}}{I_{L_{ss}}} = (1 - \frac{1}{e}) = 1 - I_4^{n-1}(\hat{E}_c) \left(\frac{I_2(\hat{E}_c)}{I_1(\hat{E}_c)} \right).$$

Substituting the expressions for the functions of \hat{E}_c and utilizing the conditional equations above in this expression gives:

$$\left[\frac{\frac{N^2 R_L}{R_c} - 1}{\frac{N^2 R_L}{R_c} + 1} \right]^{n-1} = \frac{\frac{N^2 R_L}{R_c} + 1}{\frac{N^2 R_L}{R_c} e}$$

or the response time:

$$T_r = \frac{n}{2} = \frac{1}{2} \left[1 + \frac{\ln \frac{k+1}{ke}}{\ln \frac{k-1}{k+1}} \right] \quad (18)$$

where:

$$k = \frac{N^2 R_L}{R_c}.$$

R_c / N^2	$\frac{N^2 R_L}{4 R_c}$	RESPONSE TIME BY EQUATION (18)
OHMS/TURN ²	CYCLES	CYCLES
$R_L / 4$	1	1.28
$R_L / 8$	2	2.28
$R_L / 16$	4	4.24
$R_L / 32$	8	8.21
$R_L / 64$	16	16.18

Figure 9 - Calculated response time for simple series magnetic amplifier (cycles of line frequency)

Remarks

The preceding analysis has been performed with the application of only one basic assumption. This assumption is that the transformer cores saturate completely at a very low value of magnetizing current. The use of a rectified sinusoidal control voltage is hardly an assumption but rather a recognition of existing practice. The ensuing simplification of the mathematical process is so great as to allow a direct viewing of the operation of these amplifiers in all normal modes of operation.

Direct calculation of transients gives response times in good agreement with those published heretofore and the output and control currents are subject to the same "current transformer principle" observed by all investigators in this field.

During the nonconducting period the control ampere turns have a minute value dependent upon the magnetization loops. During the conducting period the control ampere turns have a value equal to the output ampere turns, except for the ampere turns designated by the magnetization loops.

The remarkable fact that the time of response of the series magnetic amplifier does not depend upon the inductance of the transformers as reactors seems to be adequately shown by this analysis.

A NEW SERIES AMPLIFIER

A Change of Function

In the previous analysis the control current during nonconduction was considered to be such a small quantity that its existence did not affect the voltage relations. During the conducting period the ampere turns in the output circuit, determined by (9), differed from

As Storm* demonstrates, the response time is a function of the resistance ratios and is quite independent of the transformer inductances $\left(T_R = \frac{N^2 R_L}{4 R_c}\right)$. Substitution

of typical values as shown in the table of Figure 9 yields resulting response times approximately one-fourth cycle higher than those predicted by Storm* from his consideration of the "apparent inductance" of the control circuit.

Average output current as a function of control voltage calculated from (16) for a laboratory amplifier is compared with the experimental data in Figure 10. The check is not good at values of output current approaching amplifier maximum output since the experimental amplifier has inductive impedance when saturated whereas the theoretical amplifier does not.

*Storm, H. F., "Series-Connected Saturable Reactor with Control Source of Comparatively Low Impedance," AIEE Technical Paper 123 (1950)

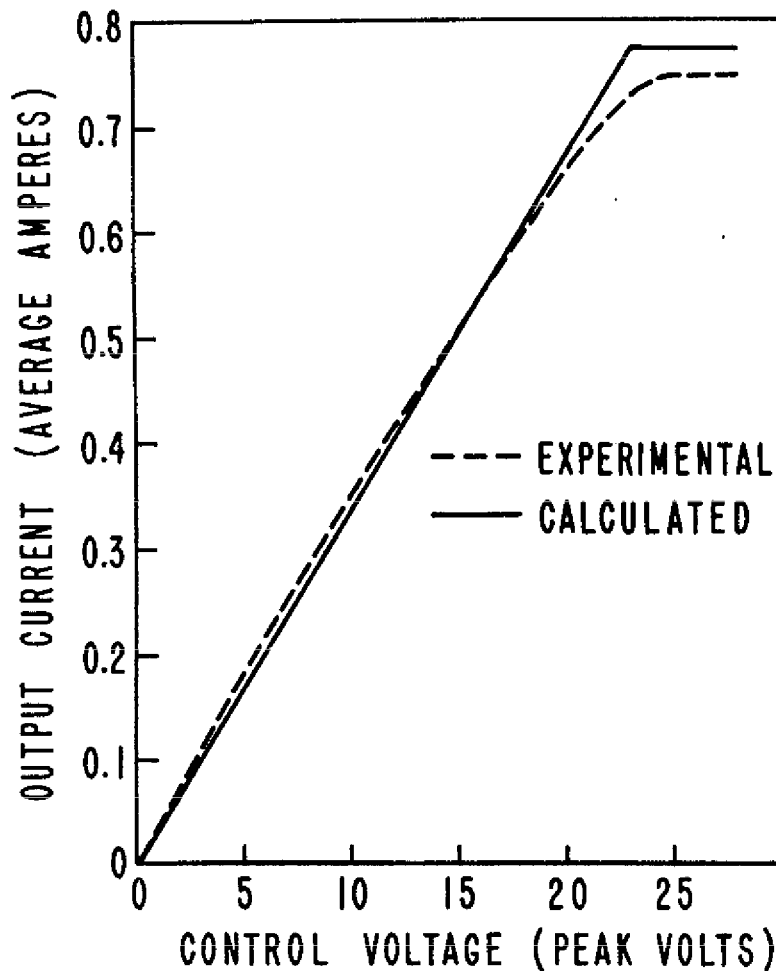


Figure 10 - Transfer characteristic, series magnetic amplifier

the control ampere turns in magnitude only by the negligibly small magnetizing ampere turns. It is readily apparent from these considerations that the major need for power from the control source occurs during the conducting period.

During the nonconducting period the flux level in the two cores is altered. One core goes through its complete flux change to saturation (10) while the other core deviates from its saturated value according to (11). During the conducting period the flux in the unsaturated core deviates further from saturation according to (12).

This section of the report will be a study of a new type of series magnetic amplifier circuit which shall be governed by the same equations as the simple magnetic amplifier during the nonconducting period but which will not include the control voltage in the equations governing the conducting period. In order to divorce the control voltage from the circuit during periods of conduction it is necessary to block the control voltage circuit during these periods. This is accomplished in the example to be analyzed by placing a rectifier "a" in series opposition to the control voltage (Figure 11). This means no

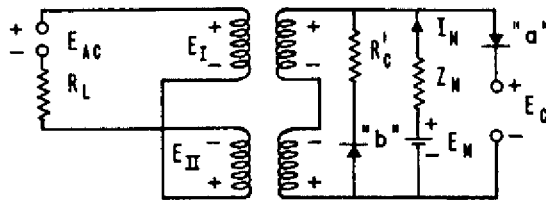


Figure 11 - New series magnetic amplifier

forward current may be carried by the control source. However, since some current is needed during nonconduction to assure operation of the constraint imposed by the control voltage, this minute current is supplied from a "constant current" source. This source, shown parallel to the control voltage circuit, consists of a high impedance Z_M and a voltage E_M of sufficient value to assure a value of current I_M as large as any current needed for the control circuit's operation.

A path for control circuit currents during the conduction periods is provided through rectifier "b" and resistance R'_c also shown parallel to the control voltage circuit.

Examination of the nonconducting period shows:

$$E_{ac} = E_I + E_{II} \quad (\text{primary circuit}) \quad (3)$$

$$NE_I = NE_{II} + E_C \quad (\text{secondary circuit}) \quad (4)$$

These are the same relations as those which governed the simple series amplifier during nonconduction.

For the conducting period (core I saturated):

$$E_{ac} = E_{II} + I_L R_L \quad (\text{primary circuit}) \quad (5)$$

$$0 = NE_{II} - I_C R'_c \quad (\text{secondary circuit}) \quad (19)$$

These are the same relations as those which governed the simple series amplifier during conduction with the exception that E_C no longer appears.

Elimination of E_{II} (5) and (19) gives:

$$E_{ac} = \left[\frac{R'_c}{N} + R_L \right] I_L \quad (20)$$

The instantaneous magnitude of output current is no longer a function of the control voltage as in (9). The control voltage may affect only the angle of firing.

Examination of the transient operation follows as in the previous section of this report. One finds the general expression defining the time of firing to be:

$$f_1(\hat{E}_C) \int_0^{t_n} \sin \omega t \, dt = f_2(\hat{E}_C) \int_0^{t_{n-1}} \sin \omega t \, dt + K \int_{t_{n-1}}^{\frac{1}{2\pi}} \sin \omega t \, dt,$$

where:

$$K = \frac{\hat{E}_{ac}}{1 + \frac{N^2 R_L}{R_c}}$$

When the definite integrals are evaluated one finds:

$$\cos \theta_n = 1 - f_1(\hat{E}_c) + f_2(\hat{E}_c) \cos \theta_{n-1} \quad (n > 1) \quad (21)$$

where:

$$f_1(\hat{E}_c) = \frac{f_2(\hat{E}_c) + K}{f_1(\hat{E}_c)}$$

$$f_2(\hat{E}_c) = \frac{f_2(\hat{E}_c) - K}{f_1(\hat{E}_c)}$$

From any set of initial conditions current firing will occur in the n th half-cycle according to (21) and will have instantaneous value determined by (20).

Using the same initial conditions and final conditions as used in the previous section of this report the transient response of a magnetic amplifier calculated for rise of output current is shown in Figure 12. The calculated response for decay of output current is shown in Figure 13.

To check the results of calculations with the equations developed above, a magnetic amplifier was set up in the laboratory and its transfer characteristics and response times predicted. Figure 14 shows a plot of the transfer characteristic obtained. The extreme closeness of the check is not significant because a mean value was chosen for the rectifier "b" resistance. This value was not sufficiently high at low output currents and caused the noted deviation. At high current the effect of the amplifier's saturated impedance has apparently compensated for the drop in rectifier resistance below the chosen mean value. Calculated and experimental values for response time checked within one-fourth cycle. The calculated values on rise and decay were 0.9 and 1.25 cycles respectively; the experimental values were approximately one cycle each.

As a further check on response calculations the response time for this experimental amplifier with doubled load circuit resistance was calculated. The predicted values were 1.25 and 2.25 cycles on rise and decay respectively; the experimental values were approximately 1.3 and 2.3 cycles for rise and decay respectively.

The schematic diagram of Figure 14 is electrically the same as that of Figure 11. Electrical isolation makes the constant current source more easily obtained.

Control Currents and "Power Gain"

Current flows through the control source only during nonconducting periods. The maximum value this current may have is determined by the constant current source.

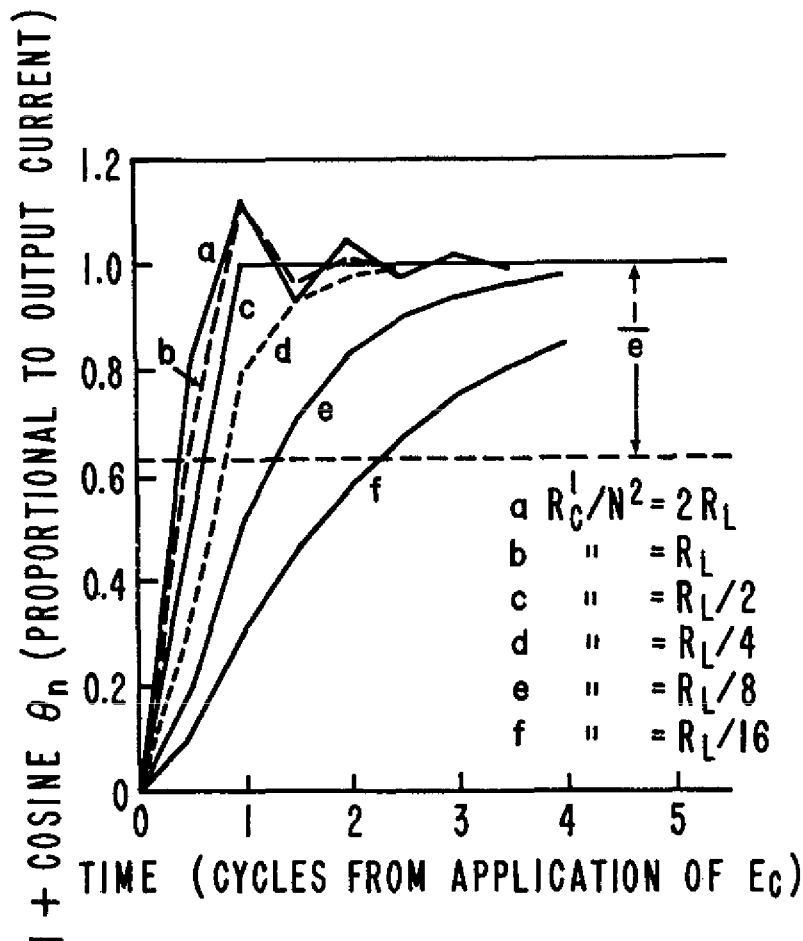


Figure 12 - Response characteristics, new series magnetic amplifier

If the magnetization loops were vertical and perfectly matched and the rectifiers had infinite inverse impedance the control current would be

$$I_c = (-) \frac{I_M \theta_n}{\pi} \quad \text{average amps} \quad (22)$$

where I_M is set experimentally at the lowest value which will assure full amplifier output.

Using the expression above, the theoretical control current for the amplifier of Figure 14 (with I_M equal to 1.02 milliamperes) was calculated and is shown in Figure 15 along with the experimental results. The character of the two curves is essentially the same. The experimental curve has smaller magnitude indicating that the practical amplifier does not quite possess the ideal characteristics noted in the theoretical calculation. The relative importance of the factors causing this deviation is the subject of further investigations.

The power amplification of this type of magnetic amplifier is a most unusual function. Discussion of this factor is complicated by the facts: (a) no power is required from the

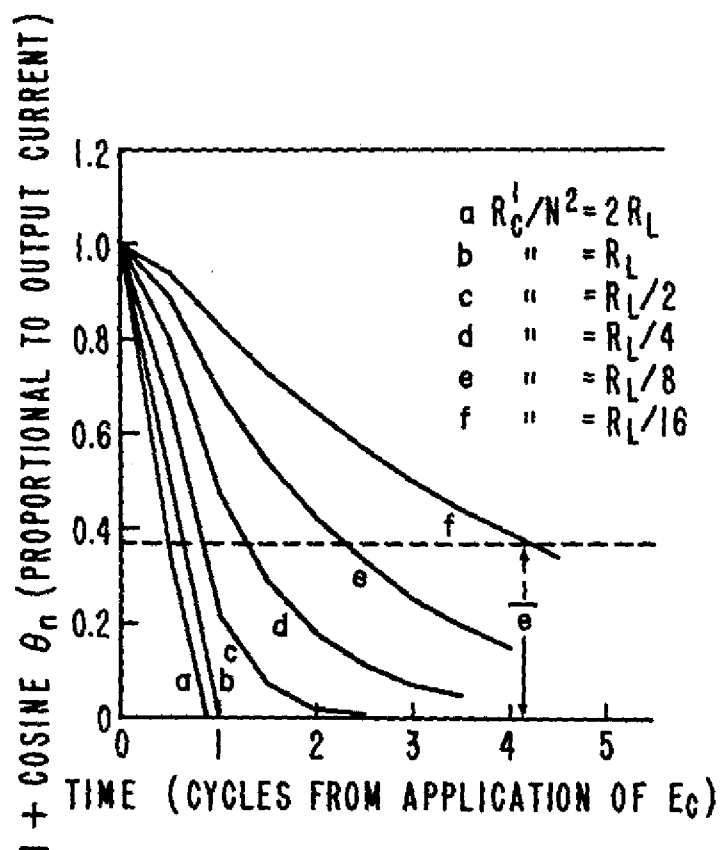
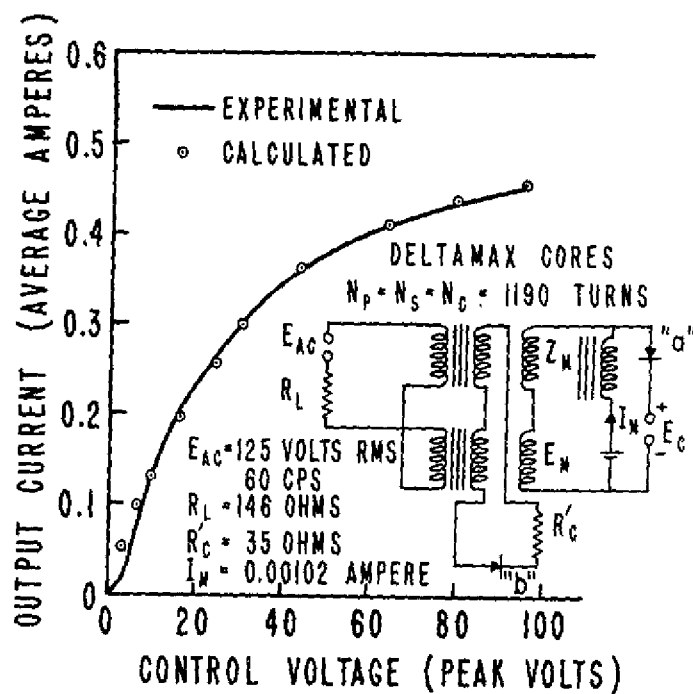


Figure 13 - Response characteristics, new series magnetic amplifier

Figure 14 - Transfer characteristic, new series magnetic amplifier



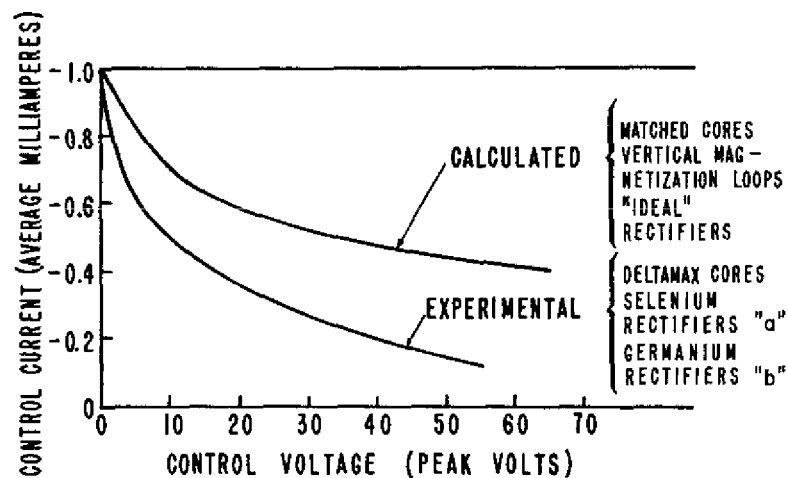


Figure 15 - Control current characteristic, new series magnetic amplifier

control source (power must be absorbed instead) and (b) the control current magnitude is an inverse function of the control voltage and can be made to become zero at maximum output power by proper choice of I_M .

If power absorbed in the control source be considered the input power and that absorbed by the load impedance the output power the "gain" of the amplifier may be examined. One finds this "gain" is a function of the control voltage with a range of "gain" values (for the practical amplifier heretofore discussed) from only a few hundred at low control voltage to infinity at maximum power output with a response time of approximately one cycle. "Gain" at one-half maximum output power was approximately 5000.

The theoretical amplifier would always have a gain of infinity since I_M could be made zero.

Remarks

One of the most interesting aspects of the preceding analysis has been the role of the control voltage. The control source is not actively engaged in the magnetization of the transformer cores—it has rather assumed the function of a passive circuit which is used for a standard or measurement reference. The power needed for the operation of the amplifier has all been drawn from the major power sources—even that power needed for the measurement of E_c . The control circuit's maximum current I_M is determined by the requirements of the cores and rectifiers and seems quite independent of the other parameters of the circuit. With appropriate materials and circuits it is readily conceivable that tremendous power gains may be obtained with a response time of one cycle.

There are two very serious disadvantages inherent in the type of magnetic amplifier discussed here. Reference to Figure 14 will show that the output currents do not bear a linear relation to the control voltage. Further, since the ac source must supply the losses associated with R_c , full use of the capabilities of the transformer cannot be attained.

The net result is that, though this type of circuit configuration exhibits fast response with high gain, it is nonlinear and has relatively low power output, for a given transformer size.

A NEW PARALLEL-CONNECTED MAGNETIC AMPLIFIER

An Obvious Improvement

Examination of the series magnetic amplifier has shown it to have serious limitations. One of the most notable arises from the fact that a change in the magnetic flux level of each core occurs during times when full output current flows in the transformer windings. Any voltage across the transformer during these periods of conduction subtracts directly from that voltage which is available for application to the load impedance. The gain is high and the response time short but the transformers are quite large for any given output and the output characteristic is far from linear.

The following section of this report will deal with an example of a type of magnetic amplifier with self-magnetizing features similar to those previously discussed but which does not require a change of magnetization in a conducting transformer.

An obvious method for eliminating the need for changing the flux level in a core whose windings are carrying output current is to allow that transformer to conduct only during alternate half-cycles and to set its magnetization level during the other half-cycles. To accomplish this, the ac supply voltage is applied to a transformer primary winding only during positive half-cycles (see Figure 16a); during negative half-cycles a rectifier "b" blocks the ac voltage from the transformer winding. In the transformer secondary or control circuit the new-familiar rectifier "a" is placed in series opposition to the control voltage which is in series with a magnetizing voltage E_z . A single winding (auto-transformer) connection would serve for illustration but would hardly seem as versatile.

The properties the magnetizing voltage E_z needs to possess are readily recognizable. During positive half-cycles E_z must prevent flow of current in the control circuit due to voltage transformed from the primary circuit (NE_{ac}) and during negative half-cycles E_z must accomplish the appropriate magnetization of the transformer core. From these considerations it is seen that E_z can be NE_{ac} .

The output current of the single-transformer magnetic amplifier is of half-wave rectified form. Full-wave rectified output is obtained by appropriate paralleling of two single-transformer amplifiers. It is this parallel amplifier with full-wave rectified output which will be the subject of the following discussion.

One example of this type of circuit is shown in Figure 16b. The primary or output circuit is the same as the well-known self-saturating bridge-type parallel magnetic amplifier. The transformer secondary terminals are connected in the same manner to the same bridge-type circuit with the exception that the load impedance is replaced by the control voltage E_c .

In the primary circuit the full load current flows in alternate half-cycles through core I and core II respectively. The existence of rectifier "b" prevents the application of the line voltage E_{ac} to the core which is not to conduct during any particular half-cycle,

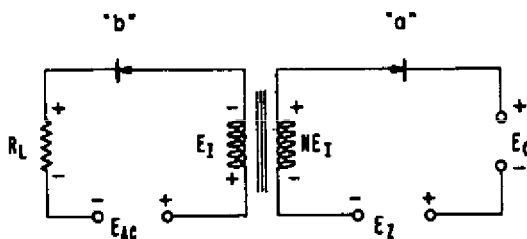


Figure 16a - Single core magnetic amplifier

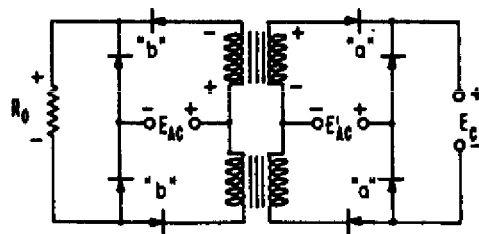


Figure 16b - New parallel magnetic amplifier

In the secondary circuit the voltages E'_{ac} is chosen to be of the same phase as the ac line voltage and of magnitude NE_{ac} . The control voltage E_c is again chosen as full-wave rectified ac of line frequency and phase and with variable amplitude.

The equations which govern the circuit during the nonconducting period (positive half-cycle) are:

$$E_{ac} = E_I$$

$$NE_{ac} - E_c = NE_{II}$$

and during negative half-cycles:

$$E_{ac} = E_{II}$$

$$NE_{ac} - E_c = NE_I$$

During periods of conduction:

$$E_{ac} = I_L R_L$$

$$NE_{ac} - E_c = NE_{II} \quad (\text{positive half-cycles})$$

$$-NE_{ac} - E_c = NE_{II} \quad (\text{negative half-cycles}).$$

This may be summarized using magnitudes as before:

- (1) Either core proceeding to saturation will have

$$|E_I| = |E_{II}| = |E_{ac}| \quad \text{volts} \quad (23)$$

across its primary terminals in the appropriate direction.

- (2) Either core deviating from its saturated state will have in both the conducting and nonconducting periods:

$$|E_I| = |E_{II}| = |E_{ac}| - \frac{|E_c|}{N} \quad \text{volts} \quad (24)$$

across its primary terminals in the appropriate direction.

(3) Either core which is saturated will be considered to have zero volts across its primary terminals.

(4) During conducting periods the output current will be:

$$I_L = \frac{E_{ac}}{R_L} \quad (25)$$

Solution of the equations above for the cosine of the angle at which firing occurs (using the same technique and with the same assumption as in the preceding analyses) gives:

$$\cos \theta_n = \frac{2|\hat{E}_c|}{N|\hat{E}_{ac}|} - 1 \quad (n > 1) \quad (26)$$

This means the firing angle for any n th half-cycle is dependent entirely upon the control voltage of the previous half-cycle. In other words, the time for full response is a half-cycle of ac voltage no matter what the other parameters of the circuit are.

The average output current would be:

$$I_L (av) = \frac{1}{\pi} \frac{|\hat{E}_{ac}|}{R_L} (1 + \cos \theta_n) = \frac{2|\hat{E}_c|}{\pi N R_L} = \frac{E_c (av)}{N R_L} \quad (27)$$

From this it is seen that the average output current is directly proportional to the average control voltage. Further, this output magnitude is independent of the ac supply voltage. Changes in the line voltage will not affect these relationships so long as the output current is below the saturation value for the amplifier setup.

Experimental Verification

An experimental amplifier using the circuit here discussed was assembled and tested. The resulting transfer characteristic is shown in Figure 17 along with the calculated ideal characteristic. The results nicely confirm the relationships predicted both in character and magnitude.

The component of R_L , due to the rectifier impedance, is a variable whose average value was determined experimentally from voltage measurements at one-half of maximum output current. The minor deviations of the experimental curve from that predicted can, to a great extent, be explained by this choice of constant rectifier impedance.

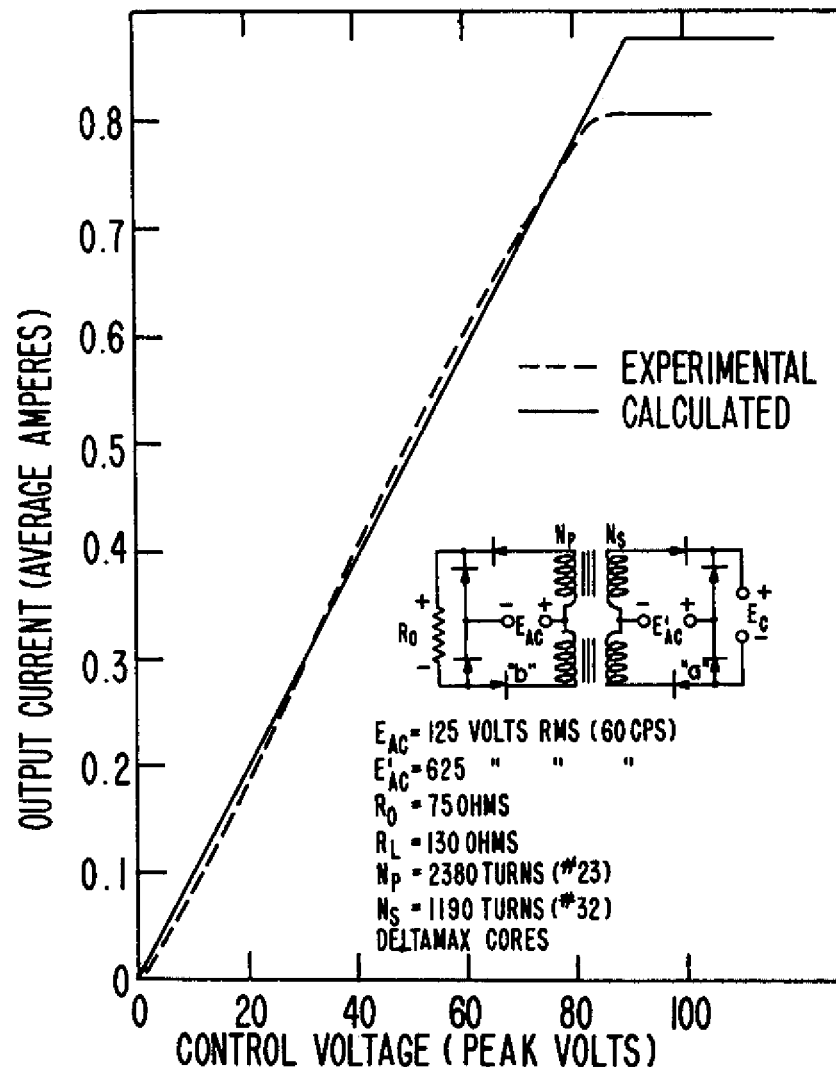


Figure 17 - Transfer characteristic, new parallel magnetic amplifier

The output current's independency of line voltage was checked experimentally also. With the control voltage set at a constant value to give one-fourth maximum output current the line voltage was reduced to 50% of nominal value with less than 10% change in output average current.

Time-of-response measurements for the experimental amplifier show the output current reaches the steady-state condition one-half cycle after application or removal of control voltage.

Control Currents and "Power Gain"

At all times during amplifier operation one of the transformer cores is being caused to deviate from its saturated condition because of the voltage relations existing in the control circuit. If the magnetization loops (Figure 1) were vertical, the cores perfectly

matched and the rectifiers ideal the current flow in the control circuit would be the constant dc value corresponding to one-half the width of the magnetization loop. This constant dc control current would flow through the control source in the direction opposite to the control voltage, i.e., the "control power" must be absorbed by the control source. If, however, a constant current of the same constant magnitude were drawn from the control source, the net current could be made ideally zero and consequently the "input power" would be zero, allowing amplifier "gain" to be infinite.

The control current for the experimental amplifier here used is shown in Figure 18 along with the ideal characteristic. Rectifier leakage in the output circuit and nonideal cores are easily seen to be the principal causes for the decrease in control current as control voltage increases. It has been found that the major difficulty arises from the rectifiers.

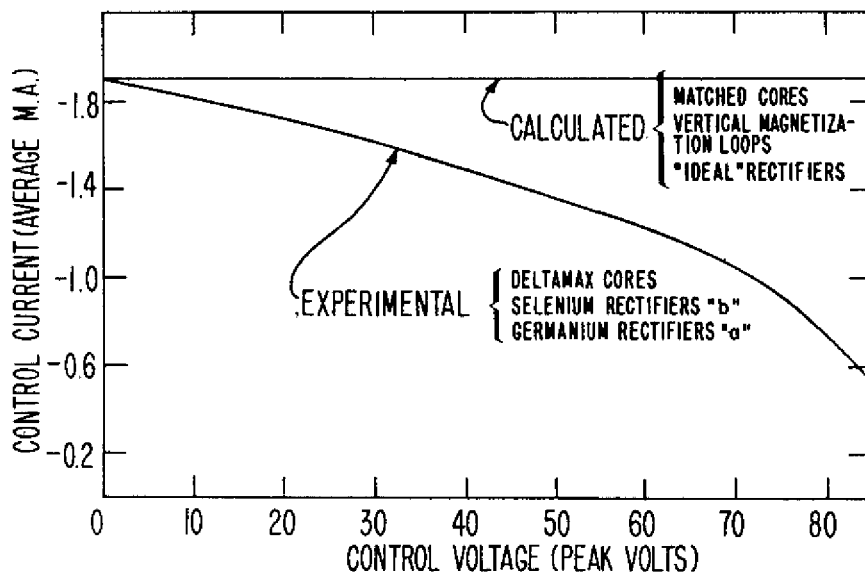


Figure 18 - Control current characteristic, new parallel magnetic amplifier

It has been experimentally determined that "gains" of more than a thousand can be obtained at 60 cycles per second (with 100% response within a cycle) using materials now abundantly available commercially and without compensating for control current. With care in selection of core materials and rectifiers, "gains" of the order of 10,000 at 60 cycles per second are possible with appropriate circuitry. The response time will remain less than one cycle. This performance is compared with today's commercially available magnetic amplifiers which, with similar response characteristics, exhibit power gains in the range of 20 to 50. Operation at higher frequencies would give increasingly better performance.

Remarks

Elimination of the necessity for magnetizing the transformer cores when a large current flows in their coils and the use of the control source as a passive element whose voltage is measured has resulted in considerable improvement in magnetic amplifier

characteristics. The obvious improvement is simultaneous availability of: short response time, high gain, good linearity, wide output range, virtual independence of supply voltage and good output power/weight ratios.

This approach to the magnetic amplifier problem has resulted from a recognition of the fact that the magnetic amplifier is a voltage-sensitive device and not, as generally believed, a current-sensitive device. The only truly independent variable is the control voltage.
